I. Définition

$$\begin{array}{lccc} & E & & F \\ f: & \mathbb{R}^n & \to & \mathbb{R}^p \\ \text{bases :} & \mathcal{B}=(e_1,\dots,e_n) & & \mathcal{B}'=(\varepsilon_1,\dots,\varepsilon_p) \\ & \vec{X}=\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} & \to & f(\vec{X}) \end{array}$$

$$\vec{X}=\sum_{j=1}^{n} x_j e_j \qquad f(\vec{X})=\sum_{j=1}^{n} x_j f(e_j) \qquad \boxed{f(e_j)=\sum_{i=1}^{p} \alpha_{i,j}\varepsilon_i}$$

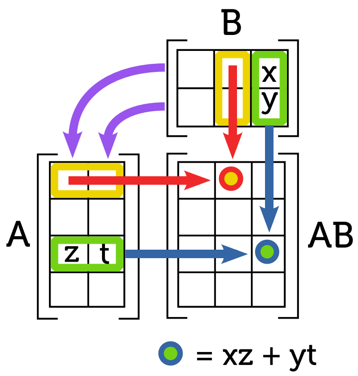
$$\boxed{M=\begin{array}{c} \begin{array}{ccccc} f(e_1) & & f(e_j) & & f(e_n) \\ \downarrow & & \downarrow & & \downarrow \end{array} \\ \begin{pmatrix} \alpha_{1,1} & \cdots & \alpha_{1,j} & \cdots & \alpha_{1,n} \\ \vdots & \ddots & \vdots & ⋰ & \vdots \\ \alpha_{i,1} & \cdots & \alpha_{i,j} & \cdots & \alpha_{i,n} \\ \vdots & ⋰ & \vdots & \ddots & \vdots \\ \alpha_{p,1} & \cdots & \alpha_{p,j} & \cdots & \alpha_{p,n} \end{pmatrix} \end{array}} \begin{array}{l} \\ \leftarrow \varepsilon_1 \\ \\ \leftarrow \varepsilon_i \\ \\ \leftarrow \varepsilon_p \end{array}$$

$$\boxed{f(\vec{X})=MX}=\begin{pmatrix} x_1\alpha_{1,1}+\cdots+x_n\alpha_{1,n} \\ \vdots \\ x_1\alpha_{p,1}+\cdots+x_n\alpha_{p,n} \end{pmatrix}$$
$$\begin{array}{ccc} \uparrow & & \uparrow \\ x_1 f(e_1) & & x_n f(e_n) \end{array}$$

II. Composée d'application

$$\boxed{M_{g\circ f}=M_g\times M_f}$$

III. Produit matriciel



IV. Dimension d'une matrice

$$\dim M_{n,p}=n\times p$$

V. Inversibilité d'une matrice de bijection

1. Théorème

$$\boxed{\underbrace{A}_{=M_f} \text{ est inversible } \Leftrightarrow \exists \underbrace{B}_{=A^{-1}=M_{f^{-1}}} \mid AB = I_n}$$

2. Méthode de calcul

a. Cas général

$$\left(A \;\vdots\; I_n \right) \Rightarrow \cdots \Rightarrow \left(I_n \;\vdots\; A^{-1} \right)$$

b. Matrices 2×2

$$A = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \qquad \boxed{A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}}$$

VI. Changement de base

Soient $\mathcal{B}(e_1, \dots, e_n)$, $\mathcal{B}'(\varepsilon_1, \dots, \varepsilon_n)$, $\mathcal{B}''$ différentes bases de E

1. Matrice de passage

$$\boxed{P = P_{\mathcal{B}\to\mathcal{B}'} = \begin{array}{c} \begin{array}{ccc} \varepsilon_1 & \dots & \varepsilon_n \\ \downarrow & & \downarrow \end{array} \\ \begin{pmatrix} \vdots & \cdots & \vdots \\ & \iddots & \\ & \cdots & \end{pmatrix} \end{array}} \begin{array}{l} \leftarrow e_1 \\ \vdots \\ \leftarrow e_n \end{array}$$

$$P_{\mathcal{B}\to\mathcal{B}''} = P_{\mathcal{B}\to\mathcal{B}'} \times P_{\mathcal{B}'\to\mathcal{B}''}$$

2. Passage de $\mathcal{B}$ à $\mathcal{B}'$

a. Vecteur

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}_{\mathcal{B}} \qquad X' = \begin{pmatrix} x'_1 \\ \vdots \\ x'_n \end{pmatrix}_{\mathcal{B}'} \qquad \boxed{X = PX'}$$

b. Matrice

$$M = \text{Mat}(f, \mathcal{B}) \qquad M' = \text{Mat}(f, \mathcal{B}') \qquad \boxed{M' = P^{-1}MP} \qquad \underline{M'^x = P^{-1}M^xP}$$